R & D NOTES

A Correlation For Laminar Free Convection Over The Entire Range Of Prandtl Numbers

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Churchill and his co-workers (Churchill and Usagi, 1972; Churchill and Ozoe, 1973; Churchill, 1977) have suggested simple empirical expressions for the heat transfer coefficient which correlate the data over the entire range of Prandtl numbers or which are valid for mixed (forced plus free) convection. The correlations proposed by these authors interpolate between two asymptotic cases: very small and very large Prandtl numbers in one kind of correlation and forced and free convection in the other kind of correlation. It is worth mentioning that the simple algebraic expressions obtained lead practically to the same results as the much more involved theoretical results (based frequently on numerical solutions). Ruckenstein (1962) and Ruckenstein and Rajagopalan (1979) have shown that an order of magnitude analysis of the governing transport equations leads to algebraic equations which can interpolate between two extremes. The two constants contained in the interpolation expression can be obtained from the appropriate values suggested by the limiting expressions. In particular, the paper of Ruckenstein and Rajagopalan (1979) provides some foundation for the interpolation equation proposed by Churchill for mixed convection. An order of magnitude evaluation is used in the present note to obtain an algebraic interpola-

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TABLE 1. CONSTANT WALL TEMPERATURE CASE—$Nu_x/Ra_x^{1/4}$ vs. Pr

Pr	Present Equation (12)	Churchill's equation Equation (14)
0.01	0.1889	0.1812
0.05	0.2771	0.2548
0.10	0.3224	0.2901
0.20	0.3687	0.3257
0.50	0.4236	0.3704
1.00	0.4549	0.4004
2.00	0.4758	0.4259
5.00	0.4911	0.4521
10.00	0.4967	0.4667
20.00	0.4997	0.4775
50.00	0.5015	0.4872
100.00	0.5021	0.4921
200.00	0.5024	0.4956
500.00	0.5026	0.4985
1 000.00	0.5026	0.5000

TABLE 2. CONSTANT WALL HEAT FLUX CASE – $Nu_x/Ra_x^{1/4}$ vs. Pr

Pr	Present Equation (13)	Churchill's equation Equation (15)
0.01	0.2176	0.2083
0.05	0.3185	0.2919
0.10	0.3696	0.3315
0.20	0.4212	0.3712
0.50	0.4810	0.4206
1.00	0.5141	0.4534
2.00	0.5358	0.4810
5.00	0.5513	0.5091
10.00	0.5570	0.5247
20.00	0.5600	0.5361
50.00	0.5618	0.5464
100.00	0.5624	0.5516
200.00	0.5627	0.5552
500.00	0.5629	0.5583
1 000.00	0.5629	0.5598

tion equation between small and large Prandtl numbers for the heat transfer coefficient for laminar free convection from a vertical plate. Both the uniform wall temperature and uniform flux cases are treated. Even though there are some differences between the presently derived expressions and those of Churchill, the numerical results given by them are near to one another.

The basic equations for laminar free convection from a vertical plate are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Because the velocity field in free convection is generated by the temperature field, a single length, δ, the thickness of the thermal boundary layer, can be taken as the length scale for both the temperature and velocity fields. The velocity scale in the x direction is denoted by u_0. Therefore, the velocity scale v_0 in the y direction is of the order of $u_0\delta/x$.

Each of the terms of Equations (2) and (3) is evaluated by replacing u and ∂u by u_0, ∂x by x, ∂y by δ, ∂T and $T - T_\infty$ by $\Delta T = T_w - T_\infty$. Consequently

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \sim \frac{u_0^2}{x} \tag{4a}$$

$$\nu\frac{\partial^2 u}{\partial y^2} \sim \frac{\nu u_0}{\delta^2}, \; a\frac{\partial^2 T}{\partial y^2} \sim \frac{a\Delta T}{\delta^2} \tag{4b}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \sim \frac{u_0 \Delta T}{x} \tag{4c}$$

If we replace each of the terms of Equations (2) and (3) by the corresponding evaluations given by Equations (4) multiplied by constants, the former equations become

$$A'\frac{u_0^2}{x} + B'\frac{\nu u_0}{\delta^2} = g\beta\Delta T \tag{5}$$

and

$$C'\frac{u_0}{x} = \frac{a}{\delta^2} \tag{6}$$

The velocity scale u_0 can be eliminated between Equations (5) and (6) to give

$$\delta^4 = \frac{a^2 x\left(A + B\dfrac{\nu}{a}\right)}{g\beta\Delta T} \tag{7}$$

which, since

$$h_x = k/\delta \tag{8}$$

transforms into

$$\frac{h_x x}{k} = \left(\frac{g\beta\Delta T x^3}{\nu a}\right)^{1/4}\left(\frac{\nu}{a}\right)^{1/4}\left(A + B\frac{\nu}{a}\right)^{-1/4} \tag{9}$$

It is worth observing that for the two limiting cases $Pr \to 0$ and $Pr \to \infty$, expression (9) acquires the forms of the exact relations given below. The constants A and B are determined from these exact expressions.

For an isothermal plate, the limiting solutions derived by LeFevre (1957) are

$$Nu_x = 0.6004\,(Ra_x Pr)^{1/4} \quad \text{for } Pr \to 0 \tag{10a}$$

$$Nu_x = 0.5027\,Ra_x^{1/4} \quad \text{for } Pr \to \infty, \tag{10b}$$

while for the uniform heat flux, the asymptotic solutions are (Churchill and Ozoe, 1973)

$$Nu_x = 0.692\,(Ra_x Pr)^{1/4} \quad \text{for } Pr \to 0 \tag{11a}$$

$$Nu_x = 0.563\,Ra_x^{1/4} \quad \text{for } Pr \to \infty \tag{11b}$$

Using the asymptotic solutions (10) and (11), we obtain

$$Nu_x = Ra_x^{1/4}\,Pr^{1/4}\,(7.6955 + 15.659\,Pr)^{-1/4} \tag{12}$$

for uniform wall temperature

and

$$Nu_x = Ra_x^{1/4} Pr^{1/4} (4.361 + 9.953 Pr)^{-1/4} \quad (13)$$

for uniform heat flux.

The corresponding empirical correlations suggested by Churchill and Ozoe (1973) to represent the data obtained by the numerical solution of the governing transport equations are

$$Nu_x = 0.503 Ra_x^{1/4} / [1 + (0.492/Pr)^{9/16})^{4/9} \quad (14)$$

for the isothermal wall and

$$Nu_x = 0.563 Ra_x^{1/4} / [1 + (0.437/Pr)^{9/16}]^{4/9} \quad (15)$$

for the uniform heat flux. Equations (12) and (14), and (13) and (15) are compared in Table 1 and 2, respectively.

The algebraic method is obviously an approximate one and must not substitute the exact solutions. It provides, however, extremely simple expressions which lead to results which are in reasonable agreement with the exact solutions.

NOTATION

a	= thermal diffusivity
A, B, A', B', C'	= numerical constants
g	= acceleration of gravity
h_x	= local heat transfer coefficient
k	= thermal conductivity
Nu_x	= $h_x x/k$ (Nusselt number)
Pr	= ν/a (Prandtl number)
Ra_x	= $g\beta\Delta T x^3/\nu a$ (Rayleigh number)
T	= temperature
T_w	= wall temperature
T_∞	= ambient temperature
ΔT	= $T_w - T_\infty$
u	= x component of velocity
u_0	= x velocity scale
v	= y component of velocity
v_0	= y velocity scale
x	= distance up the plate
y	= distance to the plate
β	= volumetric coefficient of expansion
δ	= k/h_x (thickness of the thermal boundary layer)
ν	= kinematic viscosity

LITERATURE CITED

Churchill, S. W., and R. Usagi, "A General Expression for the Correlation of Rates of Transfer and Other Phenomena," *AIChEJ.*, **18**, 1121 (1972).

Churchill, S. W., and H. Ozoe, "A Correlation for Laminar Free Convection from a Vertical Surface," Journal of Heat Transfer," *Trans. ASME, Series C*, **95**, 540 (1973).

Churchill, S. W., " A Comprehensive Correlating Equation for Laminar, Assisting, Forced and Free Convection," *AIChEJ.*, **23**, 16 (1977).

LeFevre, E. J., "Laminar Free Convection from a Vertical Plane Surface," *Proc. 9th Intern. Congr. Appl. Mech.*, Vol. 4, p. 168, Belgium (1957).

Ruckenstein, E., "Algebraic Method to Derive Equations for the Heat or Mass Transfer Coefficient," *Rev. Roumainc de Physique*, 7, 153 (1962).

———, and R., Rajagopalan, "A Simple Algebraic Method for Obtaining the Heat or Mass Transfer Coefficients under Mixed Convection," *Chem. Eng. Communications 4*, 15 (1980).

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Low Shear Viscosity of Dilute Polymer Solutions

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A modification of a viscometer originally proposed by Zimm and Crothers in 1962 is presented, which may be used to measure ultra low shear viscosity for highly dilute polymer solutions. This may provide useful information on polymer coil dimensions and relaxation time. Use of the low shear viscosity data leads to very large values of relaxation time induced by polymer addition to a concentration of only 2 to 3 wppm. This finding is consistent with the marked viscoelastic effects exhibited by these solutions.

Interest in the flow behavior of highly dilute polymer solutions has increased greatly in recent years as many potential applications of these systems have emerged. Examples include tertiary oil recovery, turbulent drag reduction and cavitation suppression. Basic to the correlation and interpretation of data for any of these applications is measurement of the solution viscosity η. A closely related quantity is the intrinsic viscosity $[\eta]$ which provides additional useful information on polymer coil dimensions and relaxation time. $[\eta]$ is obtained directly from measurement of solution viscosity η and solvent viscosity η_s by the equation

$$[\eta] = \lim_{c \to 0} \frac{\eta - \eta_s}{c\eta_s}$$

From knowledge of $[\eta]$, the primary solution relaxation time Θ, and root-mean-square end to end distance $\bar{h}$ may then be calculated (Ferry, 1970; Flory, 1969):

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